Prospects for photon blockade in four level systems in the N configuration with more than one atom.

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We show that for appropriate choices of parameters it is possible to achieve photon blockade in idealised one, two and three atom systems. We also include realistic parameter ranges for rubidium as the atomic species. Our results circumvent the doubts cast by recent discussion in the literature (Grangier *et al* Phys. Rev Lett. **81**, 2833 (1998), Imamoğlu *et al* Phys. Rev. Lett. **81**, 2836 (1998)) on the possibility of photon blockade in multi-atom systems.

I. INTRODUCTION

Recent work by Imamoğlu *et al* [1] suggested a promising scheme for observing photon blockade in a highly non-linear cavity, where the change in the Kerr nonlinearity due to single photon effects was enough to make the cavity non-resonant with modes of more than one photon. As described in their work, such a device could work as a single-photon turnstile, similar to that realised recently by Kim *et al* [2] in a semiconductor junction, which might be useful for quantum computation and the generation of non-classical light fields. The work by Imamoğlu *et al* was based on the use of the adiabatic elimination procedure and later studies [3,4] showed that the breakdown of the procedure in the high dispersion limit leads to prohibitive restrictions on the parameter space where photon blockade could be observed in a multi-atom system. Further to this, Werner and Imamoğlu [5] and Rebić *et al* [6] suggested that these problems could be overcome by using a system with a single atom. The question of observing photon blockade in multi-atom systems is of more than just theoretical interest. Possible schemes for observing photon blockade depend on directing a low flux atomic beam through a high finesse cavity and this implies that there will be an uncertainty in the number of atoms present in the cavity at any given time. We show that the photon blockade, in certain circumstances, remains strong even for a fluctuation in the atom number. Recent work [5,7] has highlighted the importance of employing a mutual detuning between the cavity and the semiclassical coupling field to shift the many atom degenerate state out of resonance. Our work builds on this idea and presents a detailed atom-cavity dressed state calculation showing the parameter regimes which offer the best prospects for the observations of photon blockade in one, two and three atom systems. We show that current technology should allow the construction of a system exhibiting photon blockade. We suggest that photon blockade can be best realised, not in a MOT as originally envisaged by Imamoğlu *et al* [1], but by sending a low atom current beam through a cavity of the style realised by Hood *et al* [8] and Münstermann *et al* [9].

Photon blockade in a cavity can be explained quite simply. An external field drives a cavity that is resonant when there are zero or one photon in the cavity and non-resonant for two (or more) photons. This can be achieved by introducing a medium into the cavity, exhibiting a large Kerr non-linearity which alters the refractive index as a function of the intensity. In general, one would expect to find large non-linearities in systems exhibiting electromagnetically induced transparency [10].

II. DRESSED STATES OF THE ATOM-CAVITY SYSTEM

Consider a four level atom as depicted in figure 1. The energy levels are labeled in order of increasing energy as $|a\rangle$, $|b\rangle$, $|c\rangle$ and $|d\rangle$ with associated energies of $\hbar\omega_a$, $\hbar\omega_b$, $\hbar\omega_c$ and $\hbar\omega_d$ respectively and transition frequencies $\omega_{\alpha\beta} = \omega_\alpha - \omega_\beta$ where $\alpha, \beta = a, b, c, d$. The $|b\rangle - |c\rangle$ transition is driven by a strong classical coupling field, with frequency $\omega_{\rm class}$, Rabi frequency Ω , detuned from the $|b\rangle - |c\rangle$ transition by an amount $\delta_{cb} = \omega_{\rm class} - \omega_{cb}$. The atoms are in a cavity with resonance frequency $\omega_{\rm cav}$ which is detuned from the $|a\rangle - |c\rangle$ transition by an amount $\delta_{ca} = \omega_{\rm cav} - \omega_{ca}$, detuned from the $|b\rangle - |d\rangle$ transition by an amount $\Delta = \omega_{\rm cav} - \omega_{db}$ and not interacting with the $|b\rangle - |c\rangle$ transition. The detunings δ_{cb} and δ_{ca} are set equal to ensure that the $|a\rangle - |b\rangle$ transition is driven by a two photon resonance. We therefore define the mutual detuning, $\delta = \delta_{cb} = \delta_{ca}$. The cavity is driven by an additional classical field with frequency $\omega_e = \omega_{\rm cav}$ and power, P. We analyse the effect of this field by examining the dressed states of the atom-cavity system. This configuration of fields and atomic levels is called the N configuration and has been considered previously [1,5,6,11]. The cavity linewidth is $\Gamma_{\rm cav}$. The atom-cavity mode coupling is $g_{\alpha_1\alpha_2} = \left(\omega_{\alpha_1\alpha_2}/2\hbar\epsilon_0 V_{\rm cav}\right)^{1/2} \mu_{\alpha_1\alpha_2}$ where α_1 and α_2 correspond to atomic levels, $\mu_{\alpha_1\alpha_2}$ is the electric dipole moment of the transition, $V_{\rm cav}$ is the cavity volume and ϵ_0 is the permittivity of free space. The Hamiltonian for the system with N atoms in the frame rotating at the cavity resonance frequency in the rotating-wave approximation is [5]

$$\begin{aligned}\frac{\widehat{\mathcal{H}}}{\hbar} &= -i\tilde{\Gamma}_c \sum_{j=1}^{N} \widehat{\sigma}^j_{cc} - i\widetilde{\Gamma}_d \sum_{j=1}^{N} \widehat{\sigma}^j_{dd} + \sum_{j=1}^{N} \Omega \left(\widehat{\sigma}^j_{cb} + \widehat{\sigma}^j_{bc}\right) \\ &\quad + \sum_{j=1}^{N} g_{ac} \left(\widehat{a}\widehat{\sigma}^j_{ca} + \widehat{a}^\dagger \widehat{\sigma}^j_{ac}\right) + \sum_{j=1}^{N} g_{bd} \left(\widehat{a}\widehat{\sigma}^j_{db} + \widehat{a}^\dagger \widehat{\sigma}^j_{bd}\right) - i\Gamma_{\rm cav}\widehat{a}^\dagger\widehat{a}. \end{aligned} \tag{1}$$

where $\tilde{\Gamma}_c = \Gamma_c + i\delta$, $\widetilde{\Gamma}_d = \Gamma_d + i\Delta$, Γ_α is the decay rate from atomic state $|\alpha\rangle$, $\widehat{a}$ $\left(\widehat{a}^\dagger\right)$ is the cavity photon annihilation (creation) operator and $\widehat{\sigma}^j_{\alpha_1\alpha_2}$ is the atomic operator $|\alpha_1\rangle\langle\alpha_2|$ acting on atom j.

We find the dressed states by diagonalizing $\widehat{\mathcal{H}}$. Fortunately, $\widehat{\mathcal{H}}$ is block diagonal in the bare state basis. The nth block can be identified by starting with the state $|a, a, \ldots, n\rangle$, representing all the atoms in state $|a\rangle$ and the cavity field in the n photon state $|n\rangle$, and finding the closed set of states coupled to $|a, a, \ldots, n\rangle$ by $\widehat{\mathcal{H}}$. Diagonalizing this block gives the n quanta manifold of dressed states.

We first consider the case of a single atom in the cavity. The zero quanta manifold consists solely of the state $|a, 0\rangle$. The one quantum manifold is spanned by the states $|a, 1\rangle$, $|b, 0\rangle$ and $|c, 0\rangle$. The corresponding block of $\widehat{\mathcal{H}}/\hbar$ can be written in matrix form in this basis as

$$\frac{\mathcal{H}^{(1)}_1}{\hbar} = \begin{bmatrix} -i\Gamma_{\rm cav} & 0 & g_{ac} \\ 0 & 0 & \Omega \\ g_{ac} & \Omega & -i\tilde{\Gamma}_c \end{bmatrix} \tag{2}$$

where the superscript on $\mathcal{H}$ refers to the number of atoms and the subscript to the number of quanta in the system. In order to simplify the expressions for eigenvalues and eigenvectors, we assume $\Gamma_{\rm cav} = 0$. The figures which follow, however, have been generated using non-zero values of $\Gamma_{\rm cav}$. Diagonalising the matrix $\mathcal{H}^{(1)}_1/\hbar$ with $\Gamma_{\rm cav} = 0$ gives the dressed state energies

$$\begin{aligned}\mathcal{E}_+ &= \left(-i\tilde{\Gamma}_c + \sqrt{-\tilde{\Gamma}_c^2 + 4\left(\Omega^2 + g_{ac}^2\right)}\right)/2 \\ \mathcal{E}_0 &= 0 \\ \mathcal{E}_- &= \left(-i\tilde{\Gamma}_c - \sqrt{-\tilde{\Gamma}_c^2 + 4\left(\Omega^2 + g_{ac}^2\right)}\right)/2\end{aligned}$$

and corresponding dressed states $|D_+\rangle$, $|D_0\rangle$ and $|D_-\rangle$ respectively. In this form, the real part of the eigenstate corresponds to the state energy and the imaginary part to the width of the state. These eigenstates form the well known Mollow triplet [12] and are presented in figure 2(a) as a function of the scaled mutual detuning, δ/Ω, with $\Gamma_c/\Omega = 0.1$, $\Gamma_{\rm cav}/\Omega = 0.01$ and $g_{ac}/\Omega = 1$. It is important to express the form of the central dressed state, $|D_0\rangle$ which is

$$|D_0\rangle = \frac{\Omega}{\sqrt{\Omega^2 + g_{ac}^2}}\,|a, 1\rangle - \frac{g_{ac}}{\sqrt{\Omega^2 + g_{ac}^2}}\,|b, 0\rangle\,.$$

Photon blockade will occur in this dressed-state picture when the cavity driving field resonantly couples the zero to one quantum manifolds, and only weakly couples the one and two quanta manifolds. We can gauge the extent of these couplings by treating each transition driven by the cavity driving field as a separate, closed two-state system. This approach will break down when multiple states are excited simultaneously. However, in situtations where the photon blockade effect occurs, the number of dressed states that are significantly occupied will be minimal and our two-state model should give a reasonably accurate picture of the degree of excitation of each transition.

The effect of the cavity driving field on the cavity-atom system can be treated by including the additional term on the right hand side of equation 1

$$\hbar\beta(\hat{a}+\hat{a}^{\dagger})$$

where $\beta=\sqrt{P\Gamma_{\mathrm{cav}}T^2/\left(4\hbar\omega_{\mathrm{cav}}\right)}$ is the external field-cavity mode coupling strength for a cavity mirror transmittance of T. In our two state model the cavity driving field drives transitions between lower and upper states, $|L\rangle$ and $|U\rangle$, in the n and $n+1$ quanta manifolds respectively. The effective Rabi frequency of the transition is given by

$$\Omega_e=\left|\beta\left\langle L\right|\hat{a}\left|U\right\rangle\right|.$$

Under these conditions, the steady state population of $|U\rangle$ is given by

$$\rho_{\mathrm{exc}}=\frac{\Omega_e^2}{2\Omega_e^2+\Delta_e^2+\Gamma_U^2}$$

where Δ_e is the detuning of the external cavity driving field from the $|L\rangle-|U\rangle$ transition and Γ_U the decay rate of state $|U\rangle$, assumed to take population from $|U\rangle$ to $|L\rangle$. Note that we have ignored the decay rate from $|L\rangle$. We denote the maximum value of ρ_{exc} over all transitions from a given lower state to all possible upper states in the n quantum manifold as $\rho_{\mathrm{exc}}^{(n)}$. For ideal photon blockade, we require $\rho_{\mathrm{exc}}^{(1)}\approx 0.5$ for the transition from the ground state, $|L\rangle=|a,a,\ldots,0\rangle=|G_0\rangle$ to the maximally coupled one quantum dressed state $|G_1\rangle$. For the case that $|G_1\rangle=|D_0\rangle$, we note that Γ_U will be small, because $|D_0\rangle$ contains no proportion of atomic state $|c\rangle$. Thus it is possible to inject a single quantum of energy into the atom-cavity system for modest values of β. Ideal blockade also requires that $\rho_{\mathrm{exc}}^{(2)}$ be negligible for transitions between $|L\rangle=|G_1\rangle$ and states $|U\rangle$ of the two quantum manifold.

We next consider the two atom, one quantum manifold of states. In this case, the basis states are $|a,a,1\rangle$, $|a,b,0\rangle$, $|a,c,0\rangle$, $|b,a,0\rangle$ and $|c,a,0\rangle$ and the corresponding block of $\widehat{\mathcal{H}}/\hbar$ can be written in matrix form as

$$\frac{\mathcal{H}_1^{(2)}}{\hbar}=\begin{bmatrix}-i\Gamma_{\mathrm{cav}} & 0 & g_{ac} & 0 & g_{ac}\\ 0 & 0 & \Omega & 0 & 0\\ g_{ac} & \Omega & -i\tilde{\Gamma}_c & 0 & 0\\ 0 & 0 & 0 & 0 & \Omega\\ g_{ac} & 0 & 0 & \Omega & -i\tilde{\Gamma}_c\end{bmatrix}\tag{3}$$

Diagonalising $\mathcal{H}_1^{(2)}/\hbar$ yields the eigenvalues

$$\begin{aligned}
\mathcal{E}_{+2}&=\left(-i\tilde{\Gamma}_c+\sqrt{-\tilde{\Gamma}_c^2+4\left(\Omega^2+2g_{ac}^2\right)}\right)/2\\
\mathcal{E}_{+1}&=\left(-i\tilde{\Gamma}_c+\sqrt{-\tilde{\Gamma}_c^2+4\Omega^2}\right)/2\\
\mathcal{E}_0&=0\\
\mathcal{E}_{-1}&=\left(-i\tilde{\Gamma}_c-\sqrt{-\tilde{\Gamma}_c^2+4\Omega^2}\right)/2\\
\mathcal{E}_{-2}&=\left(-i\tilde{\Gamma}_c-\sqrt{-\tilde{\Gamma}_c^2+4\left(\Omega^2+2g_{ac}^2\right)}\right)/2
\end{aligned}$$

with associated dressed states $|D_{+2}\rangle$, $|D_{+1}\rangle$, $|D_0\rangle$, $|D_{-1}\rangle$ and $|D_{-2}\rangle$. The dressed state energies are plotted in figure 2(b) for the same conditions as in figure 2(a). There are some important similarities between the spectrum of eigenstates for the one atom and two atom cases. In each case there is a state with zero energy, indicating that transitions from the zero to the one quantum manifold are possible for a cavity driving field tuned to the cavity resonance ω_{cav}. The states which are anti-crossing in each manifold are asymptotic to the lines $\mathcal{E}/\Omega=0$ and $\mathcal{E}/\Omega=\delta/\Omega$ with the point of closest approach being at $\delta/\Omega=0$. It is also important to realise that although there

are five distinct eigenstates only three of these eigenvalues will couple to the ground state of the atom cavity system, i.e. the matrix element $\langle a,0|\,\hat{a}\,|D_N\rangle$ is non-zero only for $N=0,\,\pm2$. For this reason only the optically active states $|D_{+2}\rangle$, $|D_0\rangle$ and $|D_{-2}\rangle$ are plotted in figure 2(b).

We have also solved the analogous three and four atom Hamiltonians and we summarise our results for the eigenstates in each case as

$$\begin{aligned}
\mathcal{E}_{+2} &= \left(-i\tilde{\Gamma}_c + \sqrt{-\tilde{\Gamma}_c^2 + 4\left(\Omega^2 + N g_{ac}^2\right)}\right)/2 \\
\mathcal{E}_{+1} &= \left(-i\tilde{\Gamma}_c + \sqrt{-\tilde{\Gamma}_c^2 + 4\Omega^2}\right)/2 \\
\mathcal{E}_{0} &= 0 \\
\mathcal{E}_{-1} &= \left(-i\tilde{\Gamma}_c - \sqrt{-\tilde{\Gamma}_c^2 + 4\Omega^2}\right)/2 \\
\mathcal{E}_{-2} &= \left(-i\tilde{\Gamma}_c - \sqrt{-\tilde{\Gamma}_c^2 + 4\left(\Omega^2 + N g_{ac}^2\right)}\right)/2
\end{aligned} \tag{4}$$

where $N=1,2,3,4$ indicates the number of atoms in the cavity. The degeneracies of the eigenvalues presented in equations 4 are interesting to observe, namely the $\mathcal{E}_{+2}$, $\mathcal{E}_0$ and $\mathcal{E}_{-2}$ values are all non-degenerate, whilst the $\mathcal{E}_{+1}$ and $\mathcal{E}_{-1}$ values are $(N-1)$ fold degenerate. (The latter values, $\mathcal{E}_{+1}$ and $\mathcal{E}_{-1}$ do not occur for $N=1$). The presence of the zero eigenvalue, $\mathcal{E}_0$, indicates that it is possible to inject one photon into the atom-cavity system with a cavity driving field tuned to ω_{cav}.

Now we consider the two quanta manifold for a cavity containing one atom. This manifold is spanned by the states $|a,2\rangle$, $|b,1\rangle$, $|c,1\rangle$ and $|d,0\rangle$ and the corresponding block of $\widehat{\mathcal{H}}/\hbar$ in matrix form is

$$\frac{\mathcal{H}_2^{(1)}}{\hbar} = \begin{bmatrix} -2i\Gamma_{\mathrm{cav}} & 0 & \sqrt{2}g_{ac} & 0 \\ 0 & -i\Gamma_{\mathrm{cav}} & \Omega & g_{bd} \\ \sqrt{2}g_{ac} & \Omega & -i\left(\Gamma_{\mathrm{cav}} + \tilde{\Gamma}_c\right) & 0 \\ 0 & g_{bd} & 0 & -i\widetilde{\Gamma}_d \end{bmatrix}$$

In order to observe photon blockade in such a system, it is essential that none of the eigenstates of $\mathcal{H}_2^{(1)}/\hbar$ are resonantly coupled by the cavity driving field to the occupied states of the one quantum manifold. To achieve this, we require $\rho_{\mathrm{exc}}^{(2)} \ll 0.5$, the smaller $\rho_{\mathrm{exc}}^{(2)}$ the greater the degree of photon blockade. A plot showing the eigenenergies of this system with associated linewidths, Γ_x where x is the dressed state under consideration, is presented in figures 3(a) and 3(b). In each case $\Delta/\Omega = 2$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 0.1$, $\Gamma_{\mathrm{cav}}/\Omega = 0.01$, $g_{ac}/\Omega = 1$ and in figure 3(a) $g_{bd}/\Omega = 1$ and the energies are plotted as a function of δ/Ω, whereas in figure 3(b) $\delta/\Omega = 0$ and the energies are plotted as a function of g_{bd}/Ω. The important features to recognise from these two traces is the shift of the smallest magnitude energy state from zero, indicating (as highlighted in [5] and [6]) that photon blockade will indeed be possible in the one atom case.

Similarly, for the two atom case, the basis states of the two quanta manifold are $|a,a,2\rangle$, $|a,b,1\rangle$, $|a,c,1\rangle$, $|b,a,1\rangle$, $|c,a,1\rangle$, $|a,d,0\rangle$, $|b,b,0\rangle$, $|b,c,0\rangle$, $|c,b,0\rangle$, $|c,c,0\rangle$,and $|d,a,0\rangle$ where the notation is $|\text{atom 1, atom 2, cavity field}\rangle$. In matrix form, the corresponding block of the Hamiltonian is

$$\frac{\mathcal{H}_2^{(2)}}{\hbar} = \begin{bmatrix}
-2i\Gamma_{\mathrm{cav}} & 0 & \sqrt{2}g_{ac} & 0 & \sqrt{2}g_{ac} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -i\Gamma_{\mathrm{cav}} & \Omega & 0 & 0 & g_{bd} & 0 & 0 & g_{ac} & 0 & 0 \\
\sqrt{2}g_{ac} & \Omega & -i\Gamma_t & 0 & 0 & 0 & 0 & 0 & 0 & g_{ac} & 0 \\
0 & 0 & 0 & -i\Gamma_{\mathrm{cav}} & \Omega & 0 & 0 & g_{ac} & 0 & 0 & g_{bd} \\
\sqrt{2}g_{ac} & 0 & 0 & \Omega & -i\Gamma_t & 0 & 0 & 0 & 0 & g_{ac} & 0 \\
0 & g_{bd} & 0 & 0 & 0 & -i\widetilde{\Gamma}_d & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega & \Omega & 0 & 0 \\
0 & 0 & 0 & g_{ac} & 0 & 0 & \Omega & -i\widetilde{\Gamma}_c & 0 & \Omega & 0 \\
0 & g_{ac} & 0 & 0 & 0 & 0 & \Omega & 0 & -i\widetilde{\Gamma}_c & \Omega & 0 \\
0 & 0 & g_{ac} & 0 & g_{ac} & 0 & 0 & \Omega & \Omega & -i\Gamma_t & 0 \\
0 & 0 & 0 & g_{bd} & 0 & 0 & 0 & 0 & 0 & 0 & -i\widetilde{\Gamma}_d
\end{bmatrix}$$

where $\Gamma_t = \Gamma_{\mathrm{cav}} + \widetilde{\Gamma}_c$.

The energy eigenvalues of $\mathcal{H}_2^{(2)}/\hbar$ are presented in figures 4(a) and 4(b). As in the two atom, single quantum case there are optically inactive states, and these have been removed from the figures. Figure 4(a) was generated for $\Delta/\Omega = 2$, $\delta/\Omega = 0$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 0.1$, $\Gamma_{\rm cav}/\Omega = 0.01$ and $g_{ac}/\Omega = 1$. As can be seen, for this case there is a zero eigenvalue so we would expect that there would be resonant coupling from the single quantum manifold to the two quantum manifold by a cavity driving field tuned to $\omega_{\rm cav}$ and hence, photon blockade would not be observed in this case. However in figure 4(b) we illustrate the effect of introducing a small mutual detuning, δ, from state $|c\rangle$. The parameters used were $\Delta/\Omega = 2$, $\delta/\Omega = 0.5$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 0.1$, $\Gamma_{\rm cav}/\Omega = 0.01$ and $g_{ac}/\Omega = 1$. It is important to observe the shift of the smallest magnitude eigenvalue from zero. Although this shift is less than for the corresponding single atom case, it is still feasible to consider building a cavity to observe this photon blockade. This point will be elaborated on in the conclusions [13].

III. PHOTON BLOCKADE IN A REALISTIC SYSTEM

Of critical importance to the experimental observation of photon blockade in atomic systems, is the question of how to trap single or small numbers of atoms within a small, high finesse optical cavity. We know of no demonstration of continuous trapping, however recent experiments conducted by Hood *et al* [8] and Münstermann *et al* [9] have shown that it is possible to have a cavity with extremely low atom fluxes passing through it. This was achieved in [8] by allowing atoms to fall from a leaky MOT into the cavity and in [9] by directing atoms out of a MOT and into the cavity. One would expect such atomic ejections to be stochastic in nature and as a consequence, the probability of having a certain number of atoms in the cavity should follow Poissonian statistics. In order to build a device which uses photon blockade, it will therefore be necessary to ensure that significant photon blockade will be observed over as wide a range of atomic numbers as possible, otherwise the photon blockade could be lost and the performance of the device degraded.

A complication in the experimental realisation of photon blockade is that the parameters chosen in the theoretical plots shown above were assumed to be independent variables. In general this will not be the case unless great care is taken in the preparation of an experiment. Specifically, for transitions in an alkali vapour which might realise the N configuration, one would expect the dipole moments of all relevant transitions to be of the same order, with the atom-cavity coupling set solely by the cavity volume, a parameter shared by both the $|a\rangle - |c\rangle$ and $|b\rangle - |d\rangle$ transitions. One would therefore expect $g_{ac} \sim g_{bd} = g$. Also, because of the shared cavity, the detuning parameters are not independent, so if we assume that $\omega_{ca} - \omega_{db} = \delta_\omega$ then we find that $\Delta = \delta + \delta_\omega$. These extra considerations will be included in the analysis to follow, which concentrates on experimentally realisable effects rather than the general theoretical demonstration presented above. We start by investigating photon blockade using the parameters $\Gamma_{\rm cav}/\Omega = 0.01$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 0.1$, $\delta_w/\Omega = 0$ and $\beta/\Omega = 1$. For these values $\rho_{\rm exc}^{(1)} \approx 0.5$, as is expected due to the presence of the strongly absorbing state at the cavity resonance.

In figure 5 (a) we show a pseudo-colour plot of $\rho_{\rm exc}^{(2)}$ as a function of δ/Ω and g/Ω for the one atom two quanta case, where colour is indicative of the value of $\rho_{\rm exc}^{(2)}$, blue being 0 and red being 0.5. The graph clearly shows $\rho_{\rm exc}^{(2)}$ decreasing monotonically with g/Ω, indicating the effectiveness of photon blockade correspondingly increasing. This is expected, as a large g will give rise to a highly nonlinear system. Also note that $\rho_{\rm exc}^{(2)}$ increases as $|\delta/\Omega|$ increases, indicating that the photon blockade is a resonance phenomenon.

In figure 5(b) we show the analogous $\rho_{\rm exc}^{(2)}$ plot for two atoms. In accord with the preliminary results which suggested that photon blockade was not possible in the multi-atom system [3,4,6] we find $\rho_{\rm exc}^{(2)} \lesssim 0.5$ in the vicinity of $\delta/\Omega = 0$, implying that photon blockade will not be observable for these choices of parameters. However, by increasing the mutual detuning and employing a modest atom-cavity coupling, regions of strong photon blockade are observable, typified by the minimum recorded value on figure 5(b) of $\rho_{\rm exc}^{(2)} = 0.0087$.

The three atom case is shown in figure 5(c) for the same parameter regime as the one and two atom cases. The form of $\rho_{\rm exc}^{(2)}$ is similar to that of the two atom case, with the same overall structure, but narrower regions where photon blockade should be observable, this is shown by the smaller blue region of figure 5(c) than figure 5(b). Again very low values of $\rho_{\rm exc}^{(2)}$ were obtained, with a minimum recorded value of $\rho_{\rm exc}^{(2)} = 0.0070$.

To use photon blockade as a tool for new quantum devices it is necessary to identify real systems in which these effects may be observed. As an example of what may be achieved with the current state of the art, we take parameters from a recent experimental paper [8] and make some minor assumptions about how they might apply to atoms falling through a high finesse cavity. It should certainly be possible to achieve a coupling field Rabi frequency of $\Omega = 10$ MHz, external field-cavity coupling strength of $\beta = 1$MHz and Hood *et al* achieve $g = 120$ MHz and $\Gamma_{\rm cav} = 40$ MHz. For a system based on transitions in the ^{87}Rb D_2 line [14] we may assume $\delta_\omega = 6600$MHz and $\Gamma_c = \Gamma_d = 17.8$ MHz. We ignore the Zeeman magnetic sublevels since these form a simple, effective three level Λ system (for states $|a\rangle$, $|b\rangle$

and $|c\rangle$) for the situation under consideration [15]. Assuming equal rates of radiative decay, these translate into our system as $g/\Omega = 12$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 1.78$, $\Gamma_{\rm cav}/\Omega = 4$, $\delta_\omega/\Omega = 660$ and $\beta/\Omega = 0.3$. In figure 6(a) we present a plot of $\rho^{(1)}_{\rm exc}$ as a function of g/Ω and δ/Ω for one atom in the cavity, with the plots showing $\rho^{(2)}_{\rm exc}$ in (b) and (c) for one and two atoms respectively.

There are some important features to note in figures 6. In 6(a) $\rho^{(1)}_{\rm exc}$ increases monotonically with g/Ω and, for the scale used, is independant of δ/Ω. The value of $\rho^{(1)}_{\rm exc}$ is qualitatively very similar for one, two and three atoms, with the values of $\rho^{(1)}_{\rm exc}$ slightly increasing as the number of atoms increases. As an example, for $g/\Omega = 12$, $\rho^{(1)}_{\rm exc} = 0.3099$, 0.3824 and 0.4148 for one, two and three atoms respectively. Traces 6(b) and (c) show $\rho^{(2)}_{\rm exc}$ for the one and two atom cases respectively. The value of $\rho^{(2)}_{\rm exc}$ for the one atom case is very small across the entire parameter space, indicating that photon blockade should be easy to observe for a single atom. Of interest is the resonance in the vicinity of $\delta = \delta_w$ which will be discussed below. There is an extra resonance in the vicinity of $\delta = 0$ for the one atom case, although this is not present to the same extent in the two and three atom cases. For two atoms, and also for three atoms (not shown), the overall values (away from the resonances) of $\rho^{(2)}_{\rm exc}$ appear to increase with the number of atoms and are much larger than for the one atom case. Qualitatively, $\rho^{(2)}_{\rm exc}$ for two and three atoms are extremely similar, with maximum values along the line $g/\Omega = 12$ of $\rho^{(2)}_{\rm exc} = 0.2244$ and 0.3092 for two and three atoms respectively. These observations would appear to agree with the intuitive idea that photon blockade would be more difficult in multi-atom systems and that there is a qualitative difference between the single atom case and multi-atom cases.

The results for $\rho^{(2)}_{\rm exc}$ in the vicinity of $\delta/\Omega = -660$ are shown in figure 7. The significance of this region is that we have $\delta = -\delta_w$ so that the cavity is now resonant with the $|b\rangle - |d\rangle$ transition. To our knowledge, this situation has not been previously explored and the consequences it has for photon blockade are significant. In figure 7 we present $\rho^{(2)}_{\rm exc}$ for one, two and three atoms in plots (a), (b) and (c) respectively. The behaviour of $\rho^{(1)}_{\rm exc}$ shows no resonance phenomena and is described above, it is only by considering $\rho^{(2)}_{\rm exc}$ that the resonance is observed. In 7(a) we see generally small values for $\rho^{(2)}_{\rm exc}$ indicating that photon blockade should be observable. With the exception of the 'shelf' for $g/\Omega \lesssim 1$, $\rho^{(2)}_{\rm exc}$ has the a similar chevron shape to that observed in the ideal case, shown in figure 5(a), although with very much smaller values. In figures 7(b) and (c), we observe a roughly triangular region of low $\rho^{(2)}_{\rm exc}$, superimposed on the background of $\rho^{(2)}_{\rm exc}$ noted earlier. Values of $\rho^{(2)}_{\rm exc} < 0.01$ are present where $\rho^{(1)}_{\rm exc} > 0.35$ for both the two and three atom cases. This suggests that the non-linearity of the system is very large about this resonance. Further investigations of this resonance will be done with increasing number of atoms to show how robust the photon blockade will be as any relaxation of the requirement for low number of atoms will enhance the prospects for experimental verification of this effect.

IV. PHOTON BLOCKADE WITH AN OFF RESONANT CAVITY

The discussions above have used a system where the cavity is driven resonantly by the external field. This has a significant drawback in trying to realise a continuously operating device inasmuch as when there are no atoms in the cavity, the cavity will absorb photons from the external field. To overcome this using the on-resonance configuration, one must run the experiment in a pulsed mode to ensure that there are no photons in the cavity prior to atoms entering the cavity. There is, however, another alternative. This occurs when the cavity driving field is not tuned to the cavity resonance, but instead is tuned to a side resonance of the one atom dressed system. In this case there will be strong coupling between the external driving field and the cavity when there is one atom in the cavity and no coupling when there are zero atoms in the cavity. By studying the form of the dressed state eigenvalues given in equation 4 it is clear that there is a dependence of the eigenvalue with the number of atoms so that it should be possible to choose g_{ac} such that the one-quantum manifold is only coupled into when there is only one atom in the cavity. These conditions are best met for the case that both δ and Ω are small compared to g_{ac} to ensure the maximum shift in eigenvalues with number of atoms.

V. CONCLUSIONS

We have presented detailed calculations which show the parameter space where photon blockade in one, two and three atom systems should be observable. By considering transitions on the ^{87}Rb D_2 line, we have suggested that photon blockade should be observable using current state of the art technology, such as has been recently demonstrated [8].

We have pointed out that strong photon blockade should be possible when the cavity resonance is tuned to the $|b\rangle - |d\rangle$ transition, despite a large mutual detuning, δ. This resonance may relax the requirements on the energy levels of the atomic species under consideration.

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VI. FIGURES

Figure 1: Energy level diagram for the four level N system. The atoms are labelled in order of increasing energy as $|a\rangle$, $|b\rangle$, $|c\rangle$, and $|d\rangle$ with energies $\hbar\omega_a$, $\hbar\omega_b$, $\hbar\omega_c$ and $\hbar\omega_d$ respectively. A strong classical coupling field with frequency $\omega_{\rm class}$ and Rabi frequency Ω is applied to the $|b\rangle - |c\rangle$ transition and detuned from it by an amount $\delta = \omega_{cb} - \omega_{\rm class}$. The atoms are placed in a cavity with resonance frequency $\omega_{\rm cav}$ which is detuned from the $|a\rangle - |c\rangle$ transition by δ and from the $|b\rangle - |d\rangle$ transition by $\Delta = \omega_{db} - \omega_{\rm cav}$. The cavity is driven resonantly by an external classical driving field with external field-cavity coupling strength β and frequency $\omega_{\rm cav}$.

Figure 2: Eigenvalues of the one quantum manifold as a function of the scaled mutual detuning δ/Ω for one and two atoms in the cavity. In each case the parameters used were $g_{ac}/\Omega = 1$, $\Gamma_{\rm cav}/\Omega = 0.01$ and $\Gamma_c/\Omega = 0.1$. In figure 2(a) the spectrum for a single atom interacting with a single cavity photon traces out the well known Mollow triplet [12]. In figure 2(b) we present the analogous trace for two atoms instead of one where the optically inactive dressed states, $|D_{+1}\rangle$ and $|D_{-1}\rangle$ have been removed.

Figure 3: Eigenvalues of the two quanta manifold for a single atom. Parameters used were the same as those in figure 2, but with the addition of $\Gamma_d/\Omega = 0.1$ and $\Delta/\Omega = 2$. Figure 3(a) has $g_{bd}/\Omega = 1$ and the energy eigenvalues are plotted as a function of scaled mutual detuning, whilst figure 3(b) has $\delta/\Omega = 0$ and the energy eigenvalues are plotted as a function of g_{bd}/Ω.

Figure 4: Eigenvalues of the two quanta manifold for two atoms in the cavity with the parameters $\Delta/\Omega = 2$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 0.1$, $\Gamma_{\rm cav}/\Omega = 0.01$ and $g_{ac}/\Omega = 1$, as a function of g_{bd}/Ω. In 4(a) the mutual detuning $\delta/\Omega = 0$ and no photon blockade is observed, in 4(b) a small mutual detuning of $\delta/\Omega = 0.5$ is applied and the photon blockade is restored for a cavity driving field tuned to $\omega_{\rm cav}$. Note that the optically inactive states have been removed in these traces also.

Figure 5: $\rho^{(2)}_{\rm exc}$ plotted as a function of scaled detuning (δ/Ω) and scaled coupling (g/Ω) for one atom (a), two atoms (b) and three atoms (c). The colour of the plot shows the value of $\rho^{(2)}_{\rm exc}$ with blue being zero, red 0.5. The parameters chosen for these figures were $\Gamma_{\rm cav}/\Omega = 0.01$, $\Gamma_c/\Omega = \Gamma_d/\Omega = 0.1$, $\delta_w/\Omega = 0$ and $\beta/\Omega = 1$. Note that over this paramter range, $\rho^{(1)}_{\rm exc} = 0.5$.

Figure 6: Pseudo-colour plots of $\rho^{(1)}_{\rm exc}$ and $\rho^{(2)}_{\rm exc}$. The parameters chosen were those which could be expected in a realistic experiment involving ^{87}Rb atoms. These parameters were $\Gamma_c/\Omega = \Gamma_d/\Omega = 1.78$, $\Gamma_{\rm cav}/\Omega = 4$, $\delta_\omega/\Omega = 660$ and $\beta/\Omega = 0.3$. Figure (a) corresponds to $\rho^{(1)}_{\rm exc}$ for one atom, whilst (b) and (c) correspond to $\rho^{(2)}_{\rm exc}$ for one and two atoms respectively. Note that the colour scales for each figure vary according to the maximum values of $\rho_{\rm exc}$.

Figure 7: Pseudo-colour plots of $\rho^{(2)}_{\rm exc}$ in the vicinity of the resonance $\delta = -\delta_w$ for one (a), two (b) and three (c) atoms. The parameters used were the same as in figure 6.

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